Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The equation to the tangent plane to the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1\ldots\ldots(1)$$

at the point (x', y', z') is

$$\frac{x'x}{a^2}+\frac{y'y}{b^2}+\frac{z'z}{c^2}=1\ \ldots.\ (2).$$

Then

$$\frac{1}{f^2}=\frac{x'^2}{a^4}+\frac{y'^2}{b^4}+\frac{z'^2}{c^4}\ldots\ldots\ (3),$$

is the required locus, an ellipsoid concentric with (1).

Also solved by *G. B. M. ZERR*, and *ELMER SCHUYLER*.

126. Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Through any fixed point O draw two straight lines at right angles. Let one line cut a given circle at Q, the other at R. Find, by Euclidean methods, the locus of the foot of the perpendicular from O upon the chord QR. Give complete analysis and discussion. Solve also by coördinate geometry.

I. Solution (Analytical) by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let r=the radius of the given circle, a=the distance of its center from O, and take the line through O and the center for the axis of x. Then the coördinate axes being rectangular, the equation to the fixed circle is

$$(x-a)^2+y^2=r^2 \text{ or } x^2+y^2-2ax+a^2-r^2=0\ldots\ldots(1).$$

If $lx+my=1$ .....(2) be the equation to QR, the equation to the lines OQ and OR is

$$x^2+y^2-2ax(lx+my)+(a^2-r^2)(lx+my)^2=0\ldots\ldots(3), \text{ or}$$

$$[1-2al+(a^2-r^2)l^2]x^2+2[lm(a^2-r^2)-am]xy+[1+(a^2-r^2)m^2]y^2=0\ \ldots.\ (4).$$

The condition that these lines are perpendicular to each other is

$$2-2al+(a^2-r^2)(l^2+m^2)=0\ldots\ldots(5).$$

The line through O and perpendicular to (2) is $x/y=l/m$......(6).

Making (5) homogeneous by aid of (2), we have,

$$[2x^2-2ax+(a^2-r^2)]l^2+(4xy-2ay)lm+[2y^2+(a^2-r^2)]m^2=0\ldots\ldots(7).$$

l and m by (6) being proportional to x and y, (7) easily becomes

$$2x^4-2ax^3+(a^2-r^2)x^2+4x^2y^2-2axy^2+2y^4+(a^2-r^2)y^2=0, \text{ or}$$

$$2(x^2+y^2)^2-[2ax-(a^2-r^2)](x^2+y^2)=0 \ldots\ldots (8),$$

giving the two circles

$$x^2+y^2=0 \ldots\ldots (9),\ x^2+y^2-ax+\tfrac{1}{2}(a^2-r^2)=0 \ \ldots\ldots (10)$$

for the required loci.

II. Solution by J. W. YOUNG, Fellow and Assistant in Mathematics, Ohio State University, Columbus, O.; and J. SCHEFFER, A. M., Hagerstown, Md.

To solve the problem by coördinate geometry, we may proceed as follows: Let the fixed point be the origin, and let the axis of x pass through the center of the circle. Its equation is

$$x^2+y^2+2gx+c=0 \ \ldots\ldots (1).$$

Let the chord, QR, be $y=mx+a \ldots\ldots (2)$.

Make (1) homogeneous by means of the relatiou $(y-mx)/a=1$. The resulting equation, viz.,

$$x^2+y^2+2gx\frac{(y-mx)}{a}+\frac{c(y-mx)^2}{a^2}=0 \ \ldots\ldots (3),$$

represents the lines through O, and Q and R.

(3) may be written $(a^2-2agm+cm^2)x^2+(a^2+c)y^2+2(ag-cm)xy=0$.

But the lines OQ and OR are at right angles. The condition for this is

$$(a^2-2agm+cm^2)+(a^2+c)=0, \text{ or } 2a^2+cm^2-2agm+c=0 \ldots\ldots (4).$$

The perpendicular from O and QR is $y=-(1/m)x \ldots\ldots (5)$.

We must find the locus of the intersection of (2) and (5), under the condition (4). (5) gives $m=-x/y$. (2) gives $a=y-mx=(y^2+x^2)/y$.

Substitute for m and a in (4), and we have

$$2\frac{(y^2+x^2)^2}{y^2}+c\left(\frac{x^2}{y^2}+1\right)+2g\frac{(x^2+y^2)x}{y^2}=0, \text{ or } 2(x^2+y^2)+2gx+c=0,$$

the required locus. This is evidently a circle, with center half way between O and the center of the given circle.